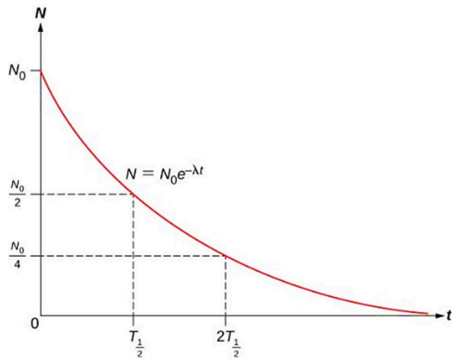
# RADIOACTIVITY



## DECAY LAW

No is the number of radioactive atoms/Nuclei at t=0
N is the number of radioactive atoms/Nuclei left after time t
Larger the value of N larger the decay
Let dN be the number of decays in time dt




$$dN\ \alpha\ -N\,dt$$
$$dN = -\lambda\,N\,dt$$

where λ is constant of proportionality called the decay constant.
The negative sign shows that the change in nuclei dN is negative.

$$\frac{dN}{N} = -\lambda\,dt$$

$$Integrating\ we\ get, \int_{No}^{N}\frac{dN}{N} = -\lambda\int_{0}^{t}dt$$

$$[\ln N]_{No}^{N} = -\lambda[t]_0^t$$

$$\ln N - \ln No = -\lambda\,[t-0]$$

$$\ln\frac{N}{No} = -\lambda t$$

$$N = Noe^{-\lambda t}$$

This is the decay law or law of radioactive decay.

## ACTIVITY

Activity A, is rate of disintegration or disintegrations per unit time

$$A = -\frac{dN}{dt} = \lambda\,N = \lambda\,No\,e^{-\lambda t}$$

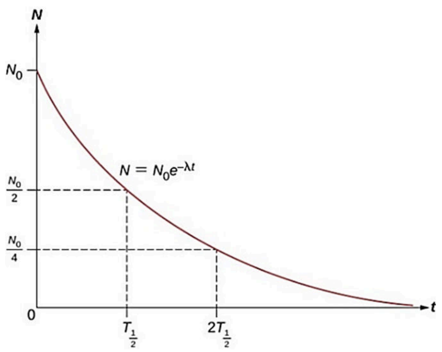
$$A = Ao\,e^{-\lambda t}\ where\ Ao = \lambda No$$

Activity is measure in becquerel (Bq) in SI unit
One becquerel is one disintegration per second
1 Curie (Ci) = $3.7 \times 10^{10}$ Bq

## HALF LIFE

The time taken for the number of parent radioactive nuclei of a oarticular species to reduce to half its value is called its half – life ($T_{½}$)

$$N = No\,e^{-\lambda t}$$
$$Put\ N\ as \frac{No}{2}$$
$$\frac{No}{2} = No\,e^{-\lambda T_{\frac{1}{2}}}$$
$$2 = e^{\lambda T_{\frac{1}{2}}}$$
$$\ln 2 = \lambda T_{\frac{1}{2}}$$
$$T_{1/2} = \frac{0.693}{\lambda}$$




So in time t=$T_{½}$ the radioactive parent nuclei reduce to No/2
In time t=2$T_{½}$ the radioactive parent nuclei reduce to No/4 and so on

## AVERAGE LIFE

Average or mean life = <u>Total lifetime of all nuclei</u>
Total number of nuclei in sample

$$\tau = \frac{\int_{No}^{0} t\,dN}{\int_{No}^{0} dN} = \frac{-\lambda\,No\,\int_0^{\infty} t\,e^{-\lambda t}\,dt}{[N]_{No}^{0}}$$

$$\tau = \frac{-\lambda\,No\,\int_0^{\infty} t\,e^{-\lambda t}\,dt}{[0-No]} = \lambda\int_0^{\infty} t\,e^{-\lambda t}\,dt$$

On integrating we get

$$\tau = \frac{1}{\lambda}$$

Thus, decay constant is also the reciprocal of mean or average life of the radioactive species.

## Size of Nucleus:

The size of the nucleus depends on the number of nucleons present in it (atomic number A)

$$R_X = R_oA^{\frac{1}{3}}\quad where\ R_o = 1.2\ X\ 10^{-15}\ m$$

The density ρ inside a nucleus is $\frac{4}{3}\pi R^3\rho = mA$,
$where\ m\ is\ the\ average\ mass\ of\ a\ nucleon$

Thus $\rho = \frac{3mA}{4\pi R_X^3} = \frac{3m}{4\pi R_O^3} = constant$

NOTE: density does not depend on A (atomic number) and is same for all nuclei. On substitution density is $2.3 \times 10^{17}$ kg $m^{-3}$ which is extremely large.

## Nuclear Force

This acts between protons and neutrons and is mostly attractive
Over short distances (about a few fm), the strength is much higher
Range is very small. Strength goes to zero when the distance between two nucleons are at a distance larger than few fm
The protons in the nucleus repel each other due to electrostatic force of repulsion. The nuclear forces between the nucleons counter this force. Since the nuclear force is much stronger for distances between nucleons in a typical nucleus, it overcomes the repulsive force and keeps the nucleus together.
The nuclear force

- Is the strongest force among subatomic particles
- The range is small and a few fm
- It is independent of the charge on the nucleons

## Nuclear Binding Energy:

Energy has to be supplied to the nucleus to free the nucleons. This energy is the binding energy of the nucleus.
Same amount of energy is released if we bring individual nucleons from infinity to form the nucleus.
The mass of the nucleus is smaller than the total mass of its constituent nucleons. Let M be the mass of the nucleus.
ΔM = Z.$m_p$ + N.$m_n$ – M , where N = A – Z and ΔM is called mass defect

Binding energy E = ΔM.$c^2$ = (Z.$m_p$ + N.$m_n$ – M)$c^2$

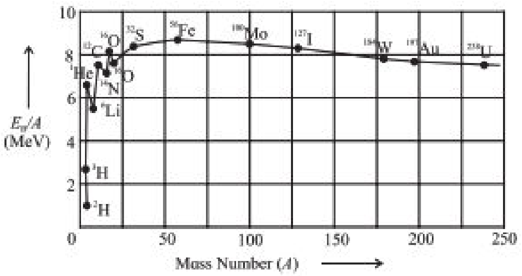
$$E = [(Z.m_P + Zm_e) + N.m_n - (M + Zm_e)]c^2$$
$$E = [Z\,m_H + Nm_n - {}_Z^AM]c^2$$

$m_H$: mass of hydrogen atom
${}_Z^AM$: mass of the element being considered

Binding energy per nucleon (=E/A) is the average energy which needs to be supplied to a nucleon to remove it from the nucleus and make it free.
**NOTE: Nuclei with higher value of E/A are more stable.**

Deuterium nucleus has the minimum value of E/A and is the least stable. The E/A value increases with increase in value of A ( atomic number) and reaches a plateau for A between 50 to 80. Thus, nuclei of these elements are most stable. Peak occurs around A=56 corresponding to iron, which is thus one of the most stable nuclei. The value of E/A then starts decreasing gradually for A > 80, making those elements slightly less stable. Biniding energy of hydrogen nucleus having one proton is Zero.


**Telephone: 9820246760 / 9769479368 • Websites: navlakhi.com –or– navlakhi.education**
**Email: abhishek@navlakhi.com –or– kunal@navlakhi.com**




**Radioactive Decay:**

The decaying nucleus if called parent nucleus which the nucleus produced after the decay is called daughter nucleus. This process is called radioactive decay or radioactivity.

***Alpha Decay:*** The parent nucleus emits an alpha particle (Helium nucleus). The parent nucleus loses 2 protons and 2 neutrons.

$^{A}_{Z}X \rightarrow\ ^{A-4}_{Z-2}Y + \alpha$

All nuclei with A>210 undergo alpha decay. Due to large number of protons the electrostatic repulsion makes the nucleus unstable and it tries to reduce the number of protons by ejecting them in the form of alpha particles.

Q-value which is the KE of the products is

$Q = [m_X - m_Y - m_{He}]\ c^2$

$^{212}_{83}Bi \rightarrow\ ^{208}_{81}Tl + \alpha$

***Beta Decay:*** Here the nucleus emits an electron produced by converting a neutron into a proton, within the nucleus.

$n \rightarrow p + e^- +$ antineutrino

Antineutrino has very little mass and no charge

The daughter nucleus, created after the decay, has one less neutron and one extra proton. Thus Z increases by 1, N increases by 1 and A remains constant.

$^{A}_{Z}X \rightarrow\ _{Z+1}^{A}Y + e^- + antineutrino$

$^{60}_{27}Co \rightarrow\ ^{60}_{28}Ni + e^- + antineutrino$

Another type if decay is the **beta plus decay** in which the proton converts to neutron by emitting a positron and a neutrino.

$p \rightarrow n + e^+ +$ neutrino

Now the mass number remains unchanged during the decay but Z increases by 1 and N increases by one.

$^{A}_{Z}X \rightarrow\ _{Z-1}^{A}Y + e^+ + neutrino$

$^{22}_{11}Na \rightarrow\ ^{22}_{10}Ne + e^+ + antineutrino$

Q-value for beta decay is

$Q = [m_X - m_Y - m_e]\ c^2$ . mass of neutrino and antineutrino is negligible.

***Gamma Decay:*** Here gamma rays, that is, highly energetic photons are emitted. Hence the daughter nucleus is same as the parent nucleus (since no particles are emitted), but has less energy.

Usually the nucleons are in the lowest possible energy state and to emit gamma radiations it needs to be excited to a high level. This is possible only if the nucleus has undergone a alpha or beta decay. Thus, gamma decay usually occurs after these decays.

**Nuclear Energy:**

It is the energy released when nuclei undergo a nuclear reaction.

Advantage: This energy is of the order of few MeV.

Disadvantage: Process is complex and expensive and can be extremely harmful

We can obtain nuclear energy by

- nuclear fission
- nuclear fusion

**Nuclear Fission:** Process in which a heavy nucleus breaks into two lighter nuclei with the release of energy is called nuclear fission.

$^{236}_{92}U$ is used for nuclear energy generation using fission. Its half life is 2.3 x $10^7$ years and activity of 6.5 X $10^{-5}$ Ci/g. However, since it fissions easily, most of its nuclei have already decayed and its not found in nature.

$^{236}_{92}U$ can be obtained from naturally occurring $^{235}_{92}U$ by bombarding it with slow neutrons.

$^{235}_{92}U + n \rightarrow\ ^{236}_{92}U$ and this $^{236}_{92}U$ can under fission in several ways producing different pairs of daughter nuclei and generating different amounts of energy.

A nuclear reactor is an apparatus in which nuclear fission is carried out in a controlled manner to produce energy in the of heat which is then converted to electricity. $^{235}_{92}U$ is used as a fuel and slow neutron bombardment creates $^{236}_{92}U$ which then undergoes fission.

Neutrons are produced in the fission reaction of $^{236}_{92}U$. The average neutrons per reaction is about 2.7. These neutron are in turn absorbed by other $^{235}_{92}U$ nuclei , thus producing $^{236}_{92}U$ which undergo fission and produce further 2.7 neutrons per fission. This can cascade and the number of neutrons produced and hence $^{236}_{92}U$ produced can increase rapidly, leading to high amount of energy and can cause an explosion. This is called ***chain reaction***. In a nuclear reactor, methods need to be employed to stop this chain reaction from occurring and the heat produced is carried away to convert it to electricity by using turbines.

NOTE: India has 22 nuclear reactors. Largest one at Kudankulam, Tamil Nadu.

**Nuclear Fusion:** It is a process in which two nuclei fuse together to form a heavier nucleus accompanied by release of nuclear energy.

For fusion to take place, it is necessary that the two nuclei come to within 1fm of each other, so they can experience nuclear forces. It becomes very difficult to come that close due to electrostatic repulsion of the electrons. Hence the atoms are first stripped off their electrons by heating. Even then, the bare nuclei experience repulsion due to the positive charges in the nucleus. Due to very high temperatures, we can provide high KE to the nuclei, thus allowing them to overcome the electrostatic repulsion and come close to one another. With higher atomic number, we have a higher positive charge in the nucleus, hence higher repulsion, hence higher KE required and hence a higher temperature of the gas is necessary.

Nuclear fusion takes place all the time in the universe. Mostly see at the center of stars where the temperature is high enough.

The temperature of the Sun is about $10^7$ K and nuclear fusion reactions keep taking place. There is fusion of 4 hydrogen nuclei to form helium nucleus. This fusion takes several steps. The effective reaction is

$4p \rightarrow \alpha + 2\ e^- +$ neutrinos + 26.7 MeV

These reactions have been going on in the Sun since 4.5 billion years.